$\varepsilon \neq 0$, $\varepsilon^2 = 0$

The minimal non-trivial axiom. Everything follows.

Algebra

R = $k[\varepsilon]/(\varepsilon^2)$. Dual numbers. The simplest non-reduced ring.

Property	Value	Why it matters
Ideals	exactly 2: (0) and (ε)	no intermediate structure
Indecomposable modules	exactly 3	the three CM-indecomposables
K_0(D_sg)	$\mathbb{Z}/2\mathbb{Z}$	two classes: signal and noise
Ext^n(k,k)	k for all n	periodic resolution, period 1
Poincaré series	1/(1-t)	pole at t=1, minimal complexity
Shift functor	Σ^2 = Id	the Z/2Z symmetry of the singularity category
Deformation space	1-dimensional	$\eta^2 = 3\eta$ is the universal deformation

The Derivative

$f(a + b\varepsilon) = f(a) + f'(a)b\varepsilon$. Exact. No approximation. No higher terms. ε IS automatic differentiation. The first derivative is exact and the second derivative does not exist. The information content of a probe with ε is precisely first order.

The Tangent Space

The Zariski tangent space T_p = Hom(m/m^2, k). With m = (ε), m^2 = 0, so m/m^2 $\cong$ k. One-dimensional. One direction. One way to leave the singular point. The embedding dimension is 2 (the split node k × $k[\varepsilon]/(\varepsilon^2)$) but the tangent dimension is 1. The defect = 2 - 1 = 1 = the syndrome dimension.

The Kähler Differentials

$\Omega^1(R/k) = R\cdot d\varepsilon$ with relation $\varepsilon\cdot d\varepsilon = 0$ (differentiate $\varepsilon^2 = 0$). The differential form exists but is torsion: position (ε) and direction $(d\varepsilon)$ are entangled. At the singular point: zero momentum. This IS confinement.

Information Theory

The Fisher information metric on the dual-number parameter space has exactly one invariant: $I(\theta) = E[(\partial \log p / \partial\theta)^2]$. No higher-order correction exists ($\varepsilon^2 = 0$). The Cramér-Rao bound is tight. The estimator IS efficient. The Hamming code IS perfect. The code sits on the Fisher manifold of dual numbers.

The Deformation

$\varepsilon^2 = 0$ over F_3 lifts to $\eta^2 = 3\eta$ over $\mathbb{Z}_3$. The deformation space is 1-dimensional ($\dim \mathrm{Ext}^1 = 1$). There is exactly one direction to deform the singularity. That direction is the 3-adic direction. The equation governing the deformation is $\eta^2 = 3\eta$. The entire 3-adic lifting is FORCED by $\dim(\mathrm{Ext}^1) = 1$.

The Connection to Everything

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$\varepsilon$$^2$ = 0
  $\to$ exact first derivative (automatic differentiation)
  $\to$ three indecomposable modules (the three types)
  $\to$ K$_0$ = $\mathbb{Z}$/2$\mathbb{Z}$ (signal vs noise)
  $\to$ periodic resolution (minimal complexity)
  $\to$ 1D deformation ($\eta$$^2$ = 3$\eta$ is universal)
  $\to$ tight Cramér-Rao (perfect code)
  $\to$ entangled position-momentum (confinement)
  $\to$ Fisher information as sole invariant
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Two words: nilpotent, order two. The poorest non-trivial algebra. The richest boundary condition. No room for deviation. Everything forced.

$$\boxed{\varepsilon \neq 0, \quad \varepsilon^2 = 0}$$